

General Physics I

PHY 101

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VECTORS

Scalar- A scalar quantity is a physical quantity that can be completely described by its magnitude. Examples of physical quantities are length, time, density, charge, volume, work etc. The number of students in a class and the cost of a house are familiar scalar quantities.

Vector- To describe certain physical quantities like displacement along with the magnitude, the direction is essential. For example, consider a body moving from X to Y. XY is the displacement. On the contrary, if the body moves from Y to X, the displacement is YX. ***‘Quantities which require both magnitude and direction for their complete specification or description are called vectors’.***

Examples of vector quantities: momentum, force, torque, magnetic field acceleration, velocity etc.

Vector Representation

- Vectors are represented by an arrow pointing in the direction of the vector.
- The length of the arrow represents the magnitude of the vector; the arrow head indicates its direction.

In printed materials, vectors are often represented by bold face type, such as **A** or with designations like $\hat{A}$, $\underline{A}$, $\vec{A}$ etc.

Alternatively we represent a vector in terms of components in three Cartesian coordinates

$$\vec{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

Hence a vector $\vec{A}$ can be written as

$$\vec{A} = 8\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$$

where, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, are unit vectors which are defined as vectors of unit magnitude in the positive x, y, z, directions respectively.

Unit vectors have a magnitude of 1.

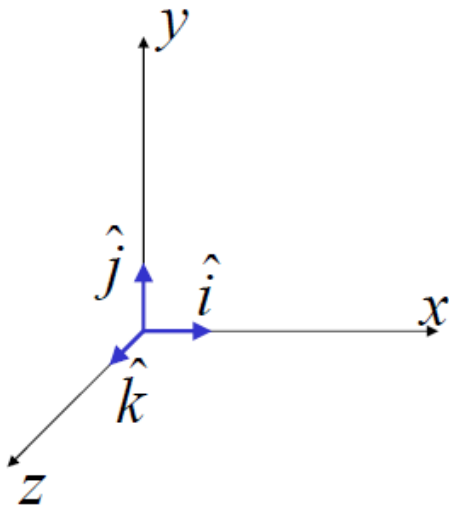
They only give the direction.

A displacement of 5 m in the x-direction is written as

$$\vec{d} = 5\mathbf{\hat{i}}$$

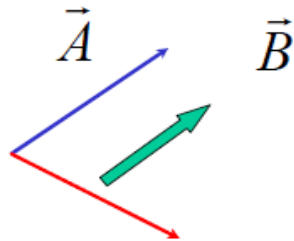
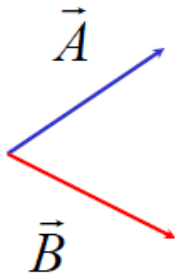
The magnitude is 5m.

The direction is the $\hat{\mathbf{i}}$ -direction.

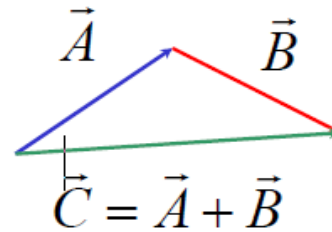


Vector Addition

Adding Vectors Graphically.

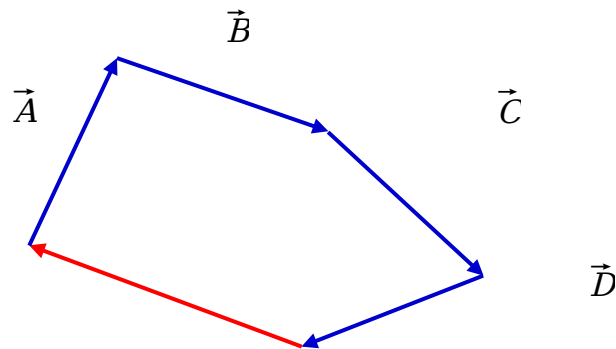


Arrange the vectors in a head to tail fashion.



The resultant is drawn from the tail of the first to the head of the last vector.

This works for any number of vectors.



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

In terms of components we can also add vectors

$$\vec{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

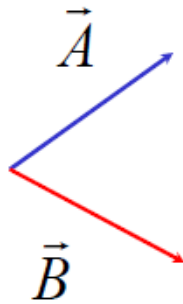
$$\vec{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

as

$$\vec{A} + \vec{B} = (A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z + B_z) \mathbf{k} \quad \dots\dots\dots (1)$$

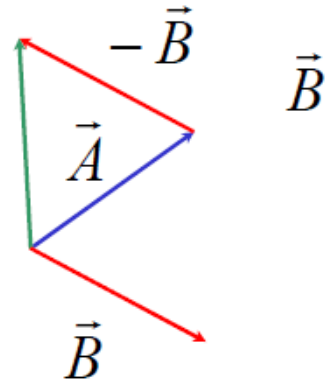
Vector Subtraction

Subtracting Vectors Graphically.



Flip one vector.

Then proceed to add the vectors



$$\vec{C} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

The resultant is drawn from the tail of the first to the head of the last vector.

In terms of components, if

$$\vec{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

and

$$\vec{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\text{Then } \vec{A} - \vec{B} = (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} + (A_z - B_z)\mathbf{k}$$

Scalar or Dot Product of two vectors:

$$\text{Given two vectors } \vec{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

and

$$\vec{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

we define the scalar or dot product of $\vec{A}$ and $\vec{B}$ as

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad \dots\dots\dots (3)$$

Alternatively, if we have diagrammatic representation of $\vec{A}$ and $\vec{B}$, then we define

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \dots\dots\dots (4)$$

where θ is the angle between vectors $\vec{A}$ and $\vec{B}$. Note that a scalar product is a scalar quantity and not a vector.

$$\hat{i} \cdot \hat{i} = 1 \quad \hat{i} \cdot \hat{j} = 0$$

$$\hat{j} \cdot \hat{j} = 1 \quad \hat{j} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{k} = 1 \quad \hat{i} \cdot \hat{k} = 0$$

A vector or cross product of two vectors:

Given two vectors, $\vec{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$

and $\vec{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$

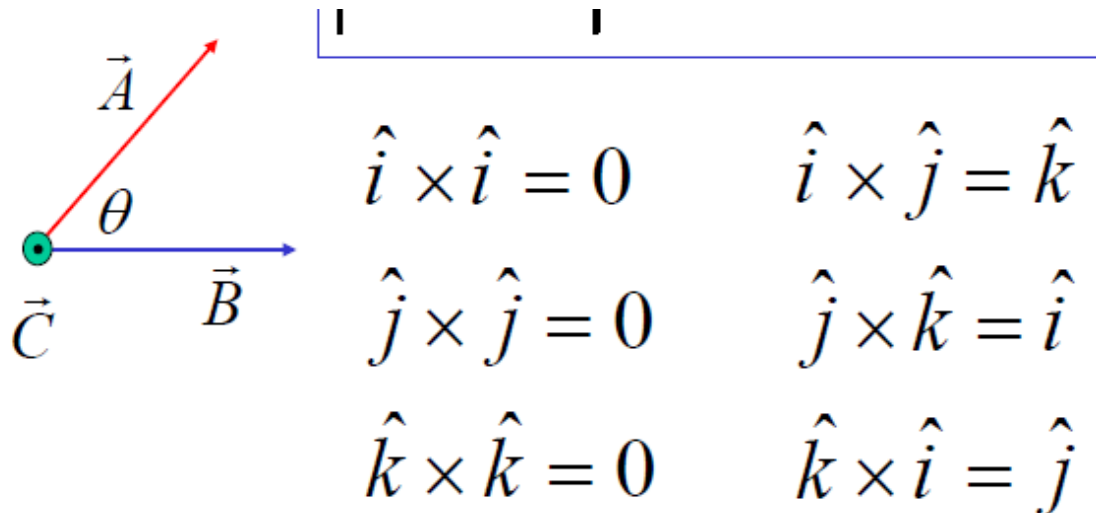
then we define a vector or cross product as

$$\begin{aligned} \vec{A} \times \vec{B} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= \mathbf{i}(A_y B_z - A_z B_y) + \mathbf{j}(A_z B_x - A_x B_z) + \mathbf{k}(A_x B_y - A_y B_x) \dots (5) \end{aligned}$$

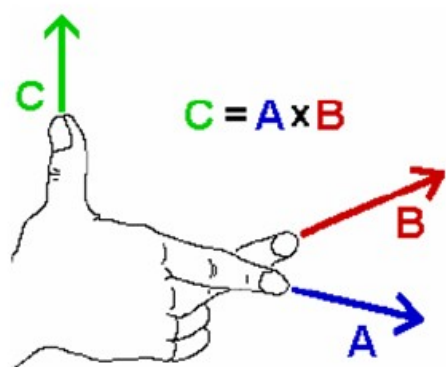
Alternatively, if we are given $\vec{A}$ and $\vec{B}$ diagrammatically

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \text{ i.e. } AB \sin \theta \dots\dots\dots (6)$$

in magnitude only with a direction defined by the motion of a right handed corkscrew when we rotate $\vec{A}$ into $\vec{B}$. Note also that vector product is a vector quantity.



Vector Multiplication II: Right Hand Rule



Index finger in the direction of the first vector.

Middle finger in the direction of the second vector

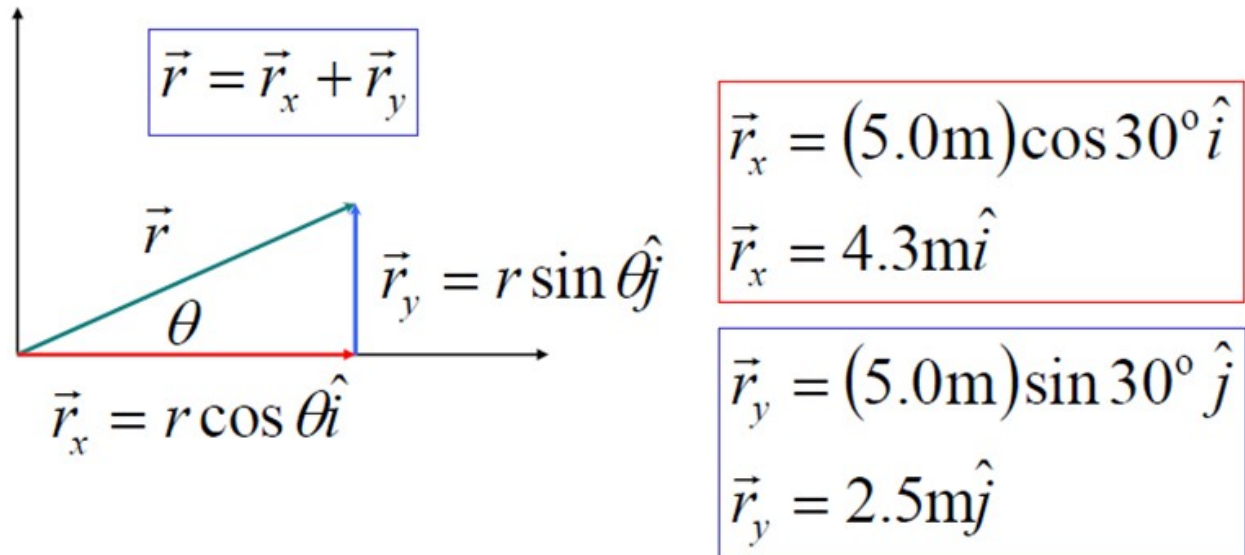
Thumb points in the direction of the cross product.

RESOLUTION OF VECTORS

Resolution of a vector means the process of splitting a vector into components. If a vector is resolved into two components along the two mutually perpendicular directions, they are called '**rectangular components**'.

Any vector can be broken down into components along the x and y axes.

Example: $\vec{r} = 5.0\text{m} @ 30^\circ$ from the horizontal. Find its components.

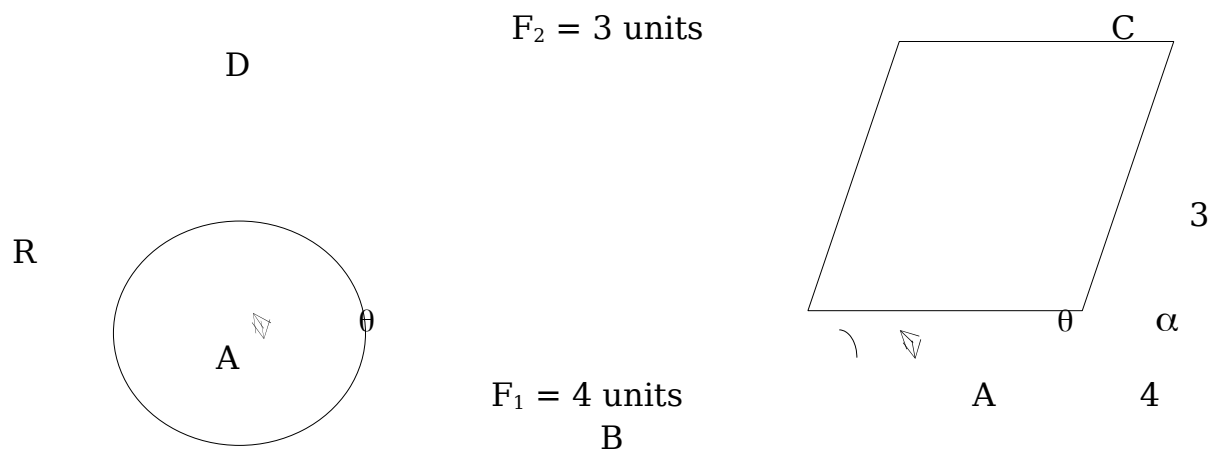


Parallelogram law of forces

The parallelogram law of forces enables us to determine the single force called the resultant which can replace the two forces acting at a point with the same effect as that of the two forces. This law states that if two forces acting simultaneously on a body at a point are presented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.

Example: In the figure below; the force $F_1 = 4$ units and force $F_2 = 3$ units are acting on a body at point A. Thus, to get the resultant of these forces, parallelogram ABCD is constructed such that $AB = 4$ units and $AC = 3$ units. Then according to

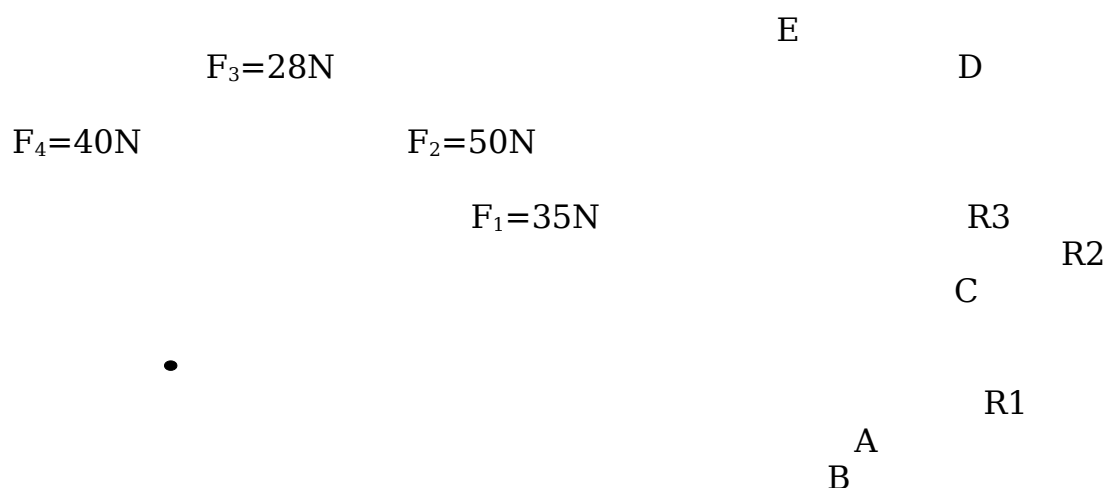
this law, the diagonal AD represents the resultant in direction and magnitude.



Hence, the resultant of the forces F_1 and F_2 on the body is equal to units corresponding to AD in the direction α to F_1 .

If more than two concurrent forces are acting on a body, two forces at a time can be combined by triangle law of forces and resultant of all the forces acting on the body may be obtained.

See the forces in the figure below:



Example 1 Given three vectors $\mathbf{A}=3\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$; $\mathbf{B}= 5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and

$$\mathbf{C} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

Find: (a) $\mathbf{A} \cdot \mathbf{B}$ (b) $\mathbf{A} \times \mathbf{B}$ (c) $(\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B}$

Solution: (a) $\mathbf{A} \cdot \mathbf{B} = (3\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}) \cdot (5\mathbf{j} + 2\mathbf{j} + 4\mathbf{k})$
 $= (3)(5) + (7)(2) + (3)(4)$
 $= 15 + 14 + 12 = 41$

(b) $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 7 & 3 \\ 5 & 2 & 4 \end{vmatrix}$

$$= \mathbf{i} \begin{vmatrix} 7 & 3 \\ 2 & 4 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 3 & 3 \\ 5 & 4 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 3 & 7 \\ 5 & 2 \end{vmatrix}$$

$$= \mathbf{i}(28 - 6) - \mathbf{j}(12 - 15) + \mathbf{k}(6 - 35)$$

$$= 22\mathbf{i} + 3\mathbf{j} - 29\mathbf{k}$$

(c) $(\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B}$

$$(\mathbf{A} \times \mathbf{C}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 7 & 3 \\ 4 & -3 & 2 \end{vmatrix}$$

$$= \mathbf{i}(14 - 9) + \mathbf{j}(12 - 6) + \mathbf{k}(-9 - 28)$$

$$= 5\mathbf{i} + 6\mathbf{j} - 37\mathbf{k}$$

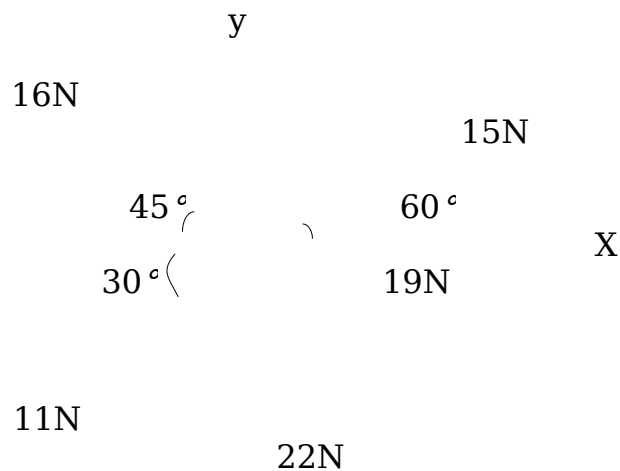
$$\therefore (\mathbf{A} \times \mathbf{C}) \cdot \mathbf{B} = (5\mathbf{i} + 6\mathbf{j} - 37\mathbf{k}) \cdot (5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$$

$$= 25 + 12 - 148$$

$$= -21$$

Example 2

The five coplanar forces shown below, act on an object. Find the resultant.



First we find the x-and y-components of each force. These components are as follows:

Force	x - component	y - component
19N	19N	0N
15N	$15\text{N}\cos 60^\circ = 7.5\text{N}$	$15\text{N}\sin 60^\circ = 13\text{N}$
16N	$-16\text{N}\cos 45^\circ = -11.3\text{N}$	$16\text{N}\sin 45^\circ = 11.3\text{N}$
11N	$-11\text{N}\cos 30^\circ = -9.53\text{N}$	$-11\text{N}\sin 30^\circ = -5.5\text{N}$
22N	0N	-22N

The resultant $\vec{R}$ has components $R_x = \sum F_x$ and

$$R_y = \sum F_y$$

Then we have

$$R_x = 19.0\text{N} + 7.5\text{N} - 11.3\text{N} - 9.53\text{N} + 0\text{N} = +5.7\text{N}$$

$$R_y = 0\text{N} + 13.0\text{N} + 11.3\text{N} - 5.5\text{N} - 22.0\text{N} = -3.2\text{N}$$

Magnitude of the resultant is

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{5.7^2 + (3.2)^2} = 6.5$$

Finally, we sketch the resultant as show in fig. b

The angle is obtained by

$$\tan \phi = \frac{3.2 N}{5.7 N} = 0.56$$

$$\therefore \phi = 29^\circ. \text{ Then } \theta = 360^\circ - 29^\circ \\ = 331^\circ.$$

The resultant is 6.5N at 331° (or -29°)

$$\vec{R} = 6.5N \text{ } - 331^\circ \text{ from } +x\text{-axis.}$$

FUNDAMENTAL LAWS OF MECHANICS

The following are the fundamental laws of mechanics:

Newton's first law

Newton's second law

Newton's third law

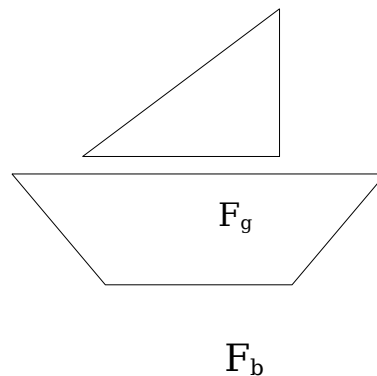
Newton's first law

It states that everybody continues in its state of rest or uniform motion in a straight line unless it is compelled by external agency acting on it. This leads to the definition of force as the external agency which changes or tends to change the state of rest or uniform linear motion of the body.

Illustration of Newton's first law:

The figure below shows a floating boat. It can be assumed that there are two forces acting on the boat. The first is the boat's weight F_g . There is also an upward buoyancy force F_b exerted

by the water on the boat. If these two forces are equal and opposite, the resultant of these two forces will be zero and therefore the boat will remain at rest (it will not move up or down).



Newton's second law:

It states that the time rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it. Thus, according to this law,

Force $\propto$ time rate of change of momentum

But momentum = mass $\times$ velocity

As mass does not change

Force $\propto$ mass $\times$ rate of change of velocity.

Force $\propto$ mass $\times$ acceleration

$\therefore F \propto ma$

If the water was removed from beneath the boat of the fig. above, a non-zero resultant force would act and the boat would accelerate downward.

Newton's Third Law:

It states that for every action, there is an equal and opposite reaction. Again considering the boat as above, the water exerts an upward buoyancy force on the boat and the boat exerts an equal and opposite force on the water. This is illustrated below:

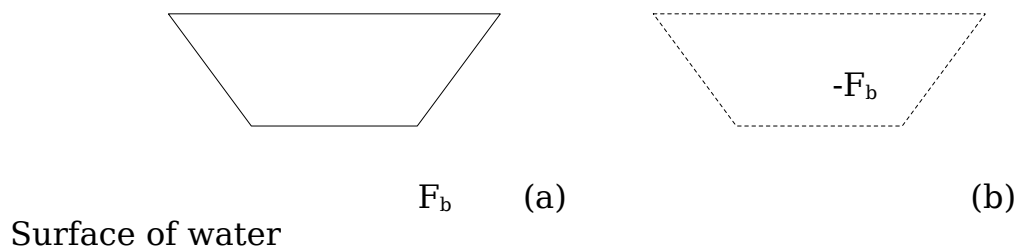


Fig: Newton's third law: (a) the water exerts a force on the boat, (b) the boat exerts an equal and opposite force on the water

Characteristics of Forces:

From Newton's 1st law, we defined the force as the agency which tries to change the state of rest or state of uniform motion of the body. From 2nd law of motion, we arrived at practical definition of unit force as the force required to produce unit acceleration in a body of 1kg mass. It may be noted that a force is completely specified only when the following four characteristics are specified:

- Magnitude
- Point of application
- Line of action

-Direction

Forces and Interactions

Interaction

*-is between one thing and another

*-requires a pair of forces acting on two objects.

Example: interaction of hand and wall pushing on each other.

Force pair-you push on wall, wall pushes on you.

Newton's Third Law of motion: Interaction

For every action, there is an equal and opposite reaction.

*in all interactions

*regardless of mass or size

The statement means that in every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs-equal and opposite action-reaction force pairs.